Logic For Free Choice Permission

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Free Choice Permission (FCP)

$$P(\varphi \vee \psi) \vDash P\varphi \wedge P\psi$$

- That you may have tee or coffee entails that you may have tea and you may have coffee.

Hilpinen's Argument (1983)

- Intensionality:

$$\text{if } \varphi \dashv\vDash \psi, \text{ then } P\varphi \vDash P\psi$$

- **Lemma**: (FCP) + (Intensionality) $\Rightarrow P\varphi \vDash P(\varphi \wedge \psi)$

  Proof. Derivation:

  1. $P\varphi$ (Assumption)
  2. $\varphi \dashv\vDash \varphi \vee (\varphi \wedge \psi)$ (Logic)
  3. $P(\varphi \vee (\varphi \wedge \psi))$ (1., 2., Intensionality)
  4. $P(\varphi \wedge \psi)$ (3., Intensionality)

- That you may have coffee entails that you may have coffee and kill the host.

Fine's Proposal (2014)

- Exact truthmaking:
  - $s \Vdash \varphi$ iff s necessitates φ's truth and is wholly relevant to it.
- Fine (2014):
  - $P\varphi$ is true iff $\forall s$(if $s \Vdash \varphi$, then s is permitted)
- Disjunction clause:
  - $s \Vdash \varphi \vee \psi$ iff $s \Vdash \varphi$ or $s \Vdash \psi$
- (Fine 2014) + (Disjunction clause) $\Rightarrow P(\varphi \vee \psi) \vDash P\varphi \wedge P\psi$
- Conjunction clause:
  - $s \Vdash \varphi \wedge \psi$ iff $\exists t, u(s = t \sqcup u,\ t \Vdash \varphi,$ and $u \vDash \psi)$
- It's possible that $s \Vdash \varphi \vee (\varphi \wedge \psi)$ but $s \nVdash \varphi$.

Our Idea

- $PP\varphi$ is true iff $\forall s$(if $s \Vdash P\varphi$, then s is permitted) $\Rightarrow s \Vdash P\varphi$?
- You may allow him to watch TV (if you want to).
- Proposal: $s \Vdash P\varphi$ iff $\forall t$(if $t \Vdash \varphi$, then s permits t)
- This leads to a natural and flexible semantic framework for permission statements.

1. Introduction

2. The Logic

3. Conclusion

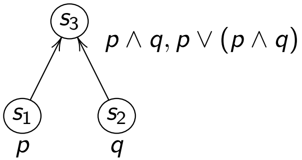
Models

- State space: $\mathcal{S} = (S, \sqsubseteq)$
  - S a non-empty set ('states')
  - $\sqsubseteq \subseteq S \times S$ ('parthood')
  - $\sqsubseteq$ is a partial order (reflexive, transitive, anti-symmetric)
  - all $s, t \in S$ have a least upper bound $s \sqcup t \in S$ ('fusion')
- Permission frame: $\mathcal{F} = (S, \sqsubseteq, R_P)$
  - $(S, \sqsubseteq)$ a state space
  - $R_P \subseteq S \times S$ ('permission').
- Model: $\mathcal{M} = (S, \sqsubseteq, R_P, v)$
  - $(S, \sqsubseteq, R_P)$ a permission frame
  - $v : \mathcal{P} \to \wp(S)$ a truthmaker assignment

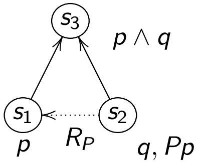
$$
\begin{array}{lll}
s \Vdash p & \text{iff} & s \in v(p) \\
s \Vdash \varphi \wedge \psi & \text{iff} & \exists t, u(s = t \sqcup u,\ t \Vdash \varphi,\ \text{and } u \vDash \psi) \\
s \Vdash \varphi \vee \psi & \text{iff} & s \Vdash \varphi \text{ or } s \Vdash \psi \\
s \Vdash P\varphi & \text{iff} & \forall t(\text{if } t \Vdash \varphi, \text{ then } sR_P t)
\end{array}
$$

Exact Entailment

- Exact entailment: $\varphi \vDash_e \psi$ iff $\forall \mathcal{M} \forall s$(if $s \Vdash \varphi$, then $s \Vdash \psi$)
- We have: $P(\varphi \vee \psi) \vDash_e P\varphi \wedge P\psi$ ☺
- We also have: $\varphi \vee (\varphi \wedge \psi) \nvDash_e \varphi$:



- In fact: $P\varphi \nvDash_e P(\varphi \wedge \psi)$



- Unfortunately also: $\varphi \wedge \psi \nvDash_e \varphi, \varphi \wedge \psi \nvDash \psi$! ☹

Classical Entailment

- Centered model: $\mathcal{M}_{@} = (S, \sqsubseteq, R_P, v, @)$
  - $\mathcal{M} = (S, \sqsubseteq, R_P, v)$ a model
  - $@ \in S$ ('actuality').
- Truth in a model:
  - $\mathcal{M}_{@} \vDash \varphi$ iff $\exists s(s \sqsubseteq @$ and $s \Vdash \varphi)$
- Classical entailment:
  - $\varphi \vDash \psi$ iff $\forall \mathcal{M}_{@}($if $\mathcal{M}_{@} \vDash \varphi$, then $\mathcal{M}_{@} \vDash \psi)$
- We now have:
  - $\varphi \wedge \psi \vDash \varphi$ and $\varphi \wedge \psi \vDash \psi$
  - $\varphi \Dashv\vDash \varphi \vee (\varphi \wedge \psi)$
  - $P(\varphi \vee \psi) \vDash P\varphi \wedge P\psi$
  - But: $P\varphi \nvDash P(\varphi \wedge \psi)$!
- ☺

Proof Theory

$$\varphi \vdash_e \varphi \qquad \frac{\varphi \vdash_e \psi}{\varphi \vdash \psi}$$

$$\frac{\varphi \vdash_{(e)} \psi \quad \psi \vdash_{(e)} \chi}{\varphi \vdash_{(e)} \chi} \qquad \varphi \vdash_e \varphi \wedge \varphi$$

$$\varphi \wedge \psi \dashv\vdash_e \psi \wedge \varphi \qquad \varphi \wedge (\psi \wedge \chi) \dashv\vdash_e (\varphi \wedge \psi) \wedge \chi$$

$$\frac{\chi \vdash_{(e)} \varphi \quad \pi \vdash_{(e)} \psi}{\chi \wedge \pi \vdash_{(e)} \varphi \wedge \psi} \qquad \varphi \wedge \psi \vdash \varphi \quad \varphi \wedge \psi \vdash \psi$$

$$\frac{\varphi \vdash_{(e)} \chi \quad \psi \vdash_{(e)} \chi}{\varphi \vee \psi \vdash_{(e)} \chi} \qquad \varphi \vdash_e \varphi \vee \psi \quad \psi \vdash_e \varphi \vee \psi$$

$$\frac{\varphi \vdash_e \psi}{P\psi \vdash_e P\varphi} \qquad \frac{\varphi \vdash_e P\psi \quad \varphi \vdash_e P\chi}{\varphi \vdash_e P(\psi \vee \chi)}$$

$$\varphi \wedge (\psi \vee \chi) \dashv\vdash_e (\varphi \wedge \psi) \vee (\varphi \wedge \chi)$$

Modularity

- Conditions on permission frames and corresponding axioms:

Condition	Axiom
$\forall s,t,u(\text{if } sR_pu \text{ and } s \sqsubseteq t, \text{ then } tR_pu)$	$P\varphi \wedge P\psi \vdash_e P(\varphi \vee \psi)$
$\forall s,t,u(\text{if } sR_pu \text{ and } t \sqsubseteq s, \text{ then } tR_pu)$	$P(\varphi \wedge \psi) \vdash_e P\varphi \wedge P\psi$
$\forall s,t,u(\text{if } sR_Pt \text{ and } tR_Pu, \text{ then } sR_Pt)$	$P\varphi \vdash_e PP\varphi$
$\vdots$	$\vdots$

- The logic with $\forall s,t,u(\text{if } sR_pu \text{ and } s \sqsubseteq t, \text{ then } tR_pu)$ is particularly pleasing:
  - $P(\varphi \vee \psi) \dashv\vDash P\varphi \wedge P\psi$
  - Define $R_P^{@} := \{s \in S : \exists t \in S(t \sqsubseteq @ \text{ and } tR_Ps)\}$:

$$\mathcal{M}_@ \vDash P\varphi \text{ iff } \forall s(\text{if } s \Vdash \varphi, \text{ then } s \in R_P^{@})$$

  - Remember: $P\varphi$ is true iff $\forall s(\text{if } s \Vdash \varphi, \text{ then } s \text{ is permitted})$

Summary and Outlook

- Using exact truthmakers, we developed a logic of permission, such that:
  - (FCP) holds
  - classical logic holds for $\vee$ and $\wedge$
  - Hilpinen's paradox is blocked because (Intensionality) fails (cf. $\varphi \vee (\varphi \wedge \psi) \not\models_e \varphi$)
  - Reasoning about permission is via exact entailment:

$$\text{if } \varphi \dashv\models_e \psi, \text{ then } P\varphi \models P\psi$$

- There are different ways of adding negation:
  - Via a prohibition relation $R_F \subseteq S \times S$:
    - (strong) $s \Vdash \neg P$ iff $\forall t(\text{if } t \Vdash \varphi, \text{ then } sR_F t)$
    - (weak) $s \Vdash \neg P$ iff $\exists t(t \Vdash \varphi \text{ and } sR_F t)$
  - Via exclusion relation $\perp \subseteq S \times S$:
    - $s \Vdash \neg P\varphi$ iff $\forall s(\text{if } s \Vdash P\varphi, \text{ then } s \perp t)$

Thanks!

Lewis's *Problem About Permission*

The problem is this. When the Master permits something, he does not thereby permit that thing to come about in whatever way the Slave pleases [...]. Suppose [...] the Master [...] says [on Thursday] to the slave ['It is permitted that the Slave does no work tomorrow']. That is all he says. He has thereby permitted a holiday, but not just any possible sort of holiday. [...] Some of the accessible worlds where the Slave does no work on Friday have been brought into permissibility, but not all of them. The Master has not said which ones. He did not need to; somehow that is understood.

(Lewis 1975)

How to Understand the Master

- The master has a *sphere of authority* over the slave, which we denote $Auth \subseteq S$.
- We have, e.g., *Slave working Friday*, *Slave working Saturday*, *Slave carrying stones*, $\ldots \in Auth$.
- φ brings ψ into permissibility iff

$$\forall s(\text{if } s \Vdash \varphi, \text{ then } \forall t(\text{if } t \Vdash \psi \text{ and } t \in Auth, \text{ then } tR_P u))$$

- P(*Slave doesn't work on Friday*) permits *Slave doesn't work on Friday* but not *Slave doesn't work on Friday and Saturday*