Logic for Free Choice Permission

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1. Free Choice Permission (FCP): $P(\varphi \vee \psi) \vDash P\varphi \wedge P\psi$ (cf. [1])
2. Intensionality: If $\varphi \dashv\vDash \psi$ (i.e. both $\varphi \vDash \psi$ and $\psi \vDash \varphi$), then $P\varphi \vDash P\psi$.
3. **Lemma** (Hilpinen [2]). If (FCP) and (Intensionality), then $P\varphi \vDash P(\varphi \wedge \psi)$.
   1. $P(\varphi)$ (Assumption)
   2. $\varphi \dashv\vDash \varphi \wedge (\varphi \wedge \psi)$ (Logic)
   3. $P(\varphi \vee (\varphi \wedge \psi))$ (1, Intensionality)
   4. $P(\varphi \wedge \psi)$ (3, FCP)
4. Exact truthmaking [4]: $s \Vdash \varphi$ iff s necessitates φ's truth and is wholly relevant to it.
5. Fine [3]: $P\varphi$ is true iff $\forall s$(if $s \Vdash \varphi$, then s is permitted)
6. State space: $\mathcal{S} = (S, \sqsubseteq)$
   - S a non-empty set ('states')
   - $\sqsubseteq \subseteq S \times S$ ('parthood')
   - $\sqsubseteq$ is a partial order (reflexive, transitive, anti-symmetric)
   - all $s, t \in S$ have a least upper bound $s \sqcup t \in S$ ('fusion')
7. Permission frame: $\mathcal{F} = (S, \sqsubseteq, R_P)$
   - $(S, \sqsubseteq)$ a state space
   - $R_P \subseteq S \times S$ ('permission').
8. Model: $\mathcal{M} = (S, \sqsubseteq, R_P, v)$
   - $(S, \sqsubseteq, R_P)$ a permission frame
   - $v : \mathcal{P} \to \wp(S)$ a truthmaker assignment

$$
\begin{array}{lll}
s \Vdash p & \text{iff} & s \in v(p) \\
s \Vdash \varphi \wedge \psi & \text{iff} & \exists t, u(s = t \sqcup u,\ t \Vdash \varphi,\ \text{and } u \vDash \psi) \\
s \Vdash \varphi \vee \psi & \text{iff} & s \Vdash \varphi \text{ or } s \Vdash \psi \\
s \Vdash P\varphi & \text{iff} & \forall t(\text{if } t \Vdash \varphi, \text{ then } sR_Pt)
\end{array}
$$

9. Exact entailment: $\varphi \vDash_e \psi$ iff $\forall \mathcal{M} \forall s$(if $s \Vdash \varphi$, then $s \Vdash \psi$)
10. Observations: $P(\varphi \vee \psi) \vDash_e P\varphi \wedge P\psi$, $\varphi \vee (\varphi \wedge \psi) \nvDash_e \varphi$, $P\varphi \nvDash_e P(\varphi \wedge \psi)$
11. Centered model: $\mathcal{M}_@ = (S, \sqsubseteq, R_P, v, @)$
    - $\mathcal{M} = (S, \sqsubseteq, R_P, v)$ a model
    - $@ \in S$ ('actuality').
12. Truth in a model: $\mathcal{M}_@ \vDash \varphi$ iff $\exists s(s \sqsubseteq @$ and $s \Vdash \varphi)$

13. **Lemma** (Truth)**.**

$$\begin{array}{lll}\mathcal{M}_@ \vDash \varphi \vee \psi & \text{iff} & \mathcal{M}_@ \vDash \varphi \text{ or } \mathcal{M}_@ \vDash \psi \\ \mathcal{M}_@ \vDash \varphi \wedge \psi & \text{iff} & \mathcal{M}_@ \vDash \varphi \text{ and } \mathcal{M}_@ \vDash \psi\end{array}$$

14. Classical entailment: $\varphi \vDash \psi$ iff $\forall \mathcal{M}_@$(if $\mathcal{M}_@ \vDash \varphi$, then $\mathcal{M}_@ \vDash \psi$)

15. Proof theory ($\varphi \dashv\vdash_{(e)} \psi$ is shorthand for both $\varphi \vdash_{(e)} \psi$ and $\psi \vdash_{(e)} \varphi$, $\varphi \vdash_{(e)} \psi$ can be uniformly disambiguated as either $\varphi \vdash_e \psi$ or $\varphi \vdash \psi$):

$$\varphi \vdash_e \varphi \qquad \frac{\varphi \vdash_e \psi}{\varphi \vdash \psi} \qquad \frac{\varphi \vdash_{(e)} \psi \quad \psi \vdash_{(e)} \chi}{\varphi \vdash_{(e)} \chi}$$

$$\varphi \vdash_e \varphi \wedge \varphi \qquad \varphi \wedge \psi \dashv\vdash_e \psi \wedge \varphi \qquad \varphi \wedge (\psi \wedge \chi) \dashv\vdash_e (\varphi \wedge \psi) \wedge \chi$$

$$\frac{\chi \vdash_{(e)} \varphi \quad \pi \vdash_{(e)} \psi}{\chi \wedge \pi \vdash_{(e)} \varphi \wedge \psi} \qquad \varphi \wedge \psi \vdash \varphi \qquad \varphi \wedge \psi \vdash \psi$$

$$\frac{\varphi \vdash_{(e)} \chi \quad \psi \vdash_{(e)} \chi}{\varphi \vee \psi \vdash_{(e)} \chi} \qquad \varphi \vdash_e \varphi \vee \psi \qquad \psi \vdash_e \varphi \vee \psi$$

$$\varphi \wedge (\psi \vee \chi) \dashv\vdash_e (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \qquad \frac{\varphi \vdash_e \psi}{P\psi \vdash_e P\varphi} \qquad \frac{\varphi \vdash_e P\psi \quad \varphi \vdash_e P\chi}{\varphi \vdash_e P(\psi \vee \chi)}$$

16. **Theorem.** $\varphi \vDash_{(e)} \psi$ iff $\varphi \vdash_{(e)} \psi$

    Proof. Via a canonical model construction and a disjunctive normal form theorem.

17. Modularity:

Condition	**Axiom**
$\forall s, t, u$(if $sR_p u$ and $s \sqsubseteq t$, then $tR_p u$)	$P\varphi \wedge P\psi \vdash_e P(\varphi \vee \psi)$
$\forall s, t, u$(if $sR_p u$ and $t \sqsubseteq s$, then $tR_p u$)	$P(\varphi \wedge \psi) \vdash_e P\varphi \wedge P\psi$
$\forall s, t, u$(if $sR_P t$ and $tR_P u$, then $sR_P t$)	$P\varphi \vdash_e PP\varphi$

18. Negation:

    - Via a prohibition relation $R_F \subseteq S \times S$:
      - (strong) $s \Vdash \neg P$ iff $\forall t$(if $t \Vdash \varphi$, then $sR_F t$)
      - (weak) $s \Vdash \neg P$ iff $\exists t(t \Vdash \varphi$ and $sR_F t)$
    - Via exclusion relation $\perp \subseteq S \times S$:
      - $s \Vdash \neg P\varphi$ iff $\forall s$(if $s \Vdash P\varphi$, then $s \perp t$)

References

[1] Hansson, Sven Ove. 2013. *The Varieties of Permission.* In *Handbook of Deontic Logic and Normative Systems*, edited Dov Gabbay, John Horty, Xavier Parent, Ron van der Meyden, and Leendert van der Torre, 195–240. *College Publications.*

[2] Hilpinen R., 1982. "Disjunctive Permission and Conditionals with Disjunctive Antecedents." In *Acta Philosophica Fennica* 35: 175–94.

[3] Fine, Kit. 2014. "Permission and Possible Worlds." In: *Dialectica* 68(3): 317–36.

[4] Fine, Kit. 2017. "Truthmaker Semantics." In: *A Companion to the Philosophy of Language*, 2nd edition, edited by Bob Hale, Crispin Wright, and Alexander Miller, 556–77. New York, NY: Wiley.