Multiple Models, One Explanation

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Abstract

Complex phenomena often require complex explanations. In simple cases, a single model can explain a phenomenon, but fully to explain a complex phenomenon, we often require systems of models. In this paper, we consider how to combine models that differ from each other along multiple lines, for instance, in the way they represent a phenomenon or the variables they include. We provide an interpretation of such situations in terms of structural realism. We focus on the conditions that allow scientists to compare different models so as to combine their results and obtain new knowledge claims. (95 words)

Keywords: Robustness analysis • multiple models • Lotka-Volterra model • structural realism

1 Introduction

In both natural and social sciences, models often do not come in isolation, but rather in sets or families. In economics, for instance, complex phenomena such as inequality, GDP growth, or recession draw on families of models that jointly contribute to our understanding of the phenomena (Aydinonat 2018). In modern physics, we have a plurality of theories and models of the same phenomena, such as different models of the atom and the atomic nucleus (Massimi 2016) or different models of the climate; in yet another field like synthetic biology, scientists integrate several methods encompassing biological data and computational analysis.

To date, literature on modelling in the philosophy of science has mainly considered the relation between one particular model and its target system, so as to explain in virtue of what a model provides a representation of its target system. In the literature, the focus has been on the characteristics of the models that justify their legitimacy as scientific tools, such as for instance that they isolate the factors that are responsible for the phenomenon of interest from negligible assumptions (Maki 1992) or that they abstract away irrelevant features (Cartwright 1989).

Less attention, however, has been paid to those cases where we have multiple models, all of which aim to explain one and the same phenomenon. In the case of multiple models, we have different, often incompatible, ways of representing the same phenomenon of interest. To account for this we need novel philosophical criteria. Several questions arise from here: How does a variety of models jointly explain the same phenomenon? What are the conditions under which we can compare models that were devised for different purposes? And what can we learn from models whose results are not consistent with each other?

In this paper, we will focus on a specific instance of many models robustness, where different models adopt different mathematical frameworks of the same target system. Our aim is to assess the extent to which they can be jointly combined to support the same hypothesis. To this purpose, in the first part of the paper we will start with an overview of *robustness analysis*—the standard method of testing whether the result of a model is invariant to slight modifications in the assumptions of the model; we will show that, by replacing individual components of a model and leaving the rest fixed, it is possible to identify what drives the final result.

While slight modifications to the initial model preserve its main components, if the differences between two models are more pronounced, we need to find a different basis for comparison. Our claim is that such a basis can be found in the underlying *structure* the models share. In support of this claim, in the third section of the paper we will draw on resources from structural realism (Worrall 1989).

Structural realism defends the view that the structure of scientific theories is preserved across theory change. In the history of science, new theories have superseded old ones and we have reasons to expect that our current best scientific theories will also be replaced by new ones in the future. In spite of the differences that scientific theories exhibit, the structural realist identifies in their shared structure the underlying component that is preserved across change. In this paper, we will argue that structural realism can be fruitfully applied not only to different theories over time, but also when several theories are all candidate explanations of the same given phenomenon.

2 Many-Models Robustness

Robustness analysis is a method of testing whether the results of a model are invariant to changes in the assumptions of the model. The reason behind this strategy is to check if the unrealistic assumptions of the model–e.g., simplifications, idealizations and abstractions–do not undermine the final results. By showing that the output is invariant under changes in the assumptions, we have reason to believe that the result does not depend on the unrealistic assumptions of the model.

More formally, suppose that we have a model M, which consists of a core block C that formulates the hypothesis under study, and a set of unrealistic assumptions $A_1, \ldots, A_n$ that represent in a simplified manner the properties of the target system. By replacing each single assumption with a different one, we can test that the result is invariant across conditions and, if so, we have an indication that the result does not depend on the specific assumptions of the model.

The procedure described above is an example of robustness analysis, which takes place at the level of the assumptions of the model. In the literature, Weisberg and Reismann (2008) call this kind of robustness "parameter robustness". Alongside parameter robustness, Weisberg and Reismann discuss another example of robustness analysis, this time taking place at the level of the variables of the model. The idea is that, by introducing new variables, it is possible to observe whether further specifications of the target system significantly affect the final result.

Finally, a third instance of robustness analysis is "representational robustness" and refers to variations in the mathematical framework in which a model is represented. An example of this case is the combination of computational and analytic approaches of the phenomenon of interest that are based on different mathematical assumptions. The aim of representational robustness is to show that the final result is invariant across conditions, irrespective of the mathematical description of the target system.

In a sense, representational robustness is the most radical test of robustness, because it combines models that fundamentally differ from each other. This is the kind of robustness with which we engage in this paper. The reason why this strategy requires a different assessment than the previous cases, is that here the final result derives from models that differ along multiple lines, e.g. for the variables they adopt and their mathematical formulation. In other words, there no longer is a correspondence between the "core" of the model that is shared across conditions and a number of interchangeable satellite assumptions. So, what is it the logic underlying this strategy?

On the one hand, it might argued that if the result is invariant across models that significantly differ from each other, then the confidence in the final result is even higher than in cases of models that only slightly differ from each other. However, if two models represent the target system in fundamentally different ways, then the logic of robustness analysis that we applied in the case of parameter or structural robustness no longer applies. In order to maintain that two models that are representationally different can nevertheless support each other, we need to do away with the view that the "core" of the model represents accurately the target system as against its unrealistic assumptions. Instead, what we suggest is that the basis for comparison can be found at a more fundamental level than the mathematical formulations of the specific models, namely at the level of the models' underlying common structure.

In order to give a more detailed example, in the next section, we will outline the analysis of representational robustness as in Weisberg and Reismann (2008). The authors discuss the Lotka-Volterra model and compare its results achieved via differential equations versus individual-based models (IBMs).

2.1 Representational Robustness

The Lotka-Volterra model is a model in population ecology that describes the relation between prey and predators in an ecological system. It consists of two coupled first-order differential equations that were discovered independently by Alfred Lotka (1925) and Vito Volterra (1926). Volterra was a mathematician interested in marine biology who arrived at the formulation of the prey-predator model to explain the following, surprising observation: during the years of WWI, when fishery was reduced in the Adriatic sea, the population of predatory fish increased relatively to the prey population. To model the prey-predator dynamics, Lotka and Volterra propose the following equations:

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

where,

$x(t) =$ prey population at t

$y(t) =$ predator population at t

$\alpha =$ intrinsic growth rate of prey

$\beta =$ capture efficiency of the predators

$\delta =$ reproduction efficiency of predators

$\gamma =$ intrinsic death rate of predators

These *Lotka-Volterra equations* have periodic solutions, which means that the population growth of the two populations oscillates regularly over time according to the two species' interactions. When the prey population grows, the predator population starts increasing as well as it has more prey to capture; the more predators there are, the more prey they kill, so that the prey growth start diminishing. A decrease in prey population determines a decrease in the predator population until the cycle of oscillations starts again from the beginning.

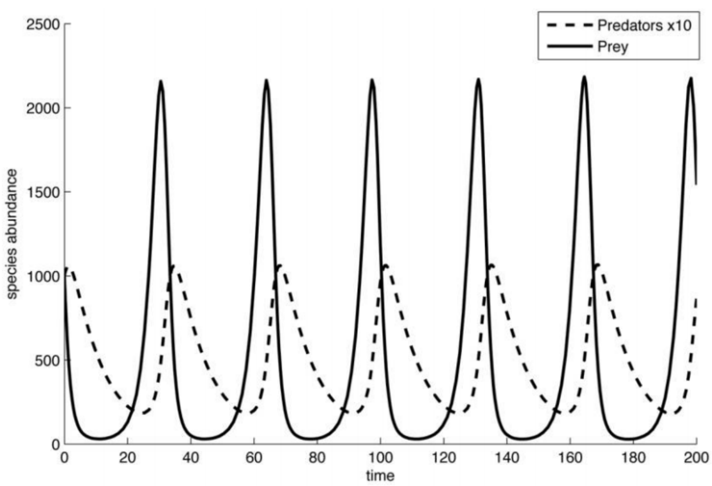


Figure 1. Lotka-Volterra model's oscillation. [1]

[1] All illustrations are taken from (Weisberg and Reismann 2008).

From the Lotka-Volterra equations it is possible to derive the *Volterra principle*, which states that an external cause of death in the system, which equally affects the prey and the predator, will determine a relative increase in the abundance of the prey population as compared to the predator population. The principle explains Volterra's surprising observation of fish catches in the Adriatic sea: by removing a cause of death in the system, i.e. fishery during WWI, the predator population relatively increases as compared to the prey population.

The Lotka-Volterra equations provide an exact solution to the prey-predator interactions but, at the same time, they describe the ecological system in a highly idealised way. In order to test whether the Volterra Principle holds true even in a less idealised setup, Weisberg and Reisman developed an individual-based model of the prey-predator system. IBMs are based on computer simulations with prey and predators populations that live according to rules of movement, reproduction, predation and death. The mathematical solutions of IBMs are different than the ones obtained via differential equations: in one case we have exact solutions, in another approximate solutions. IBM's use discrete populations, while differential equations use continuous populations. In spite of the differences, IBMs allow us to test whether the properties of the predator-prey system hold in the computational version of the Lotka-Volterra model.

As figure 2 shows, the IBMs solutions exhibit important differences as compared to the differential equations. For one thing, IBMs do not have periodic oscillations, for another one (or both) populations eventually get extinct.

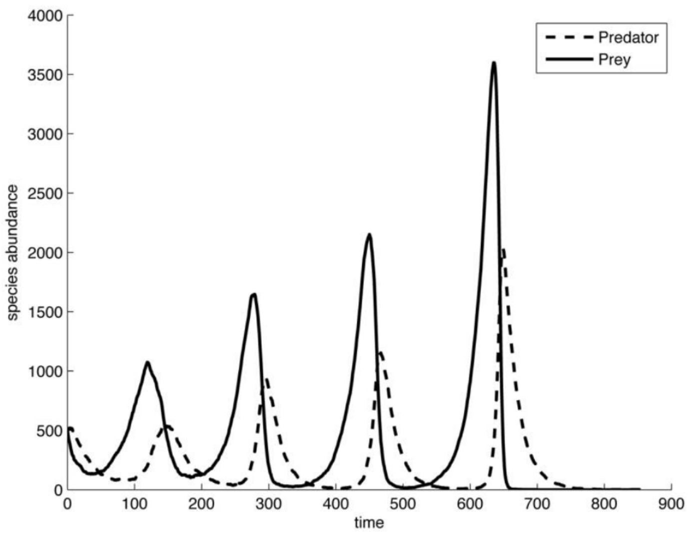
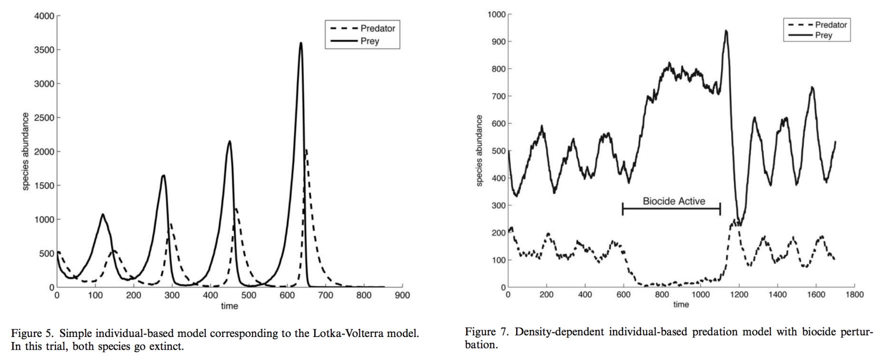
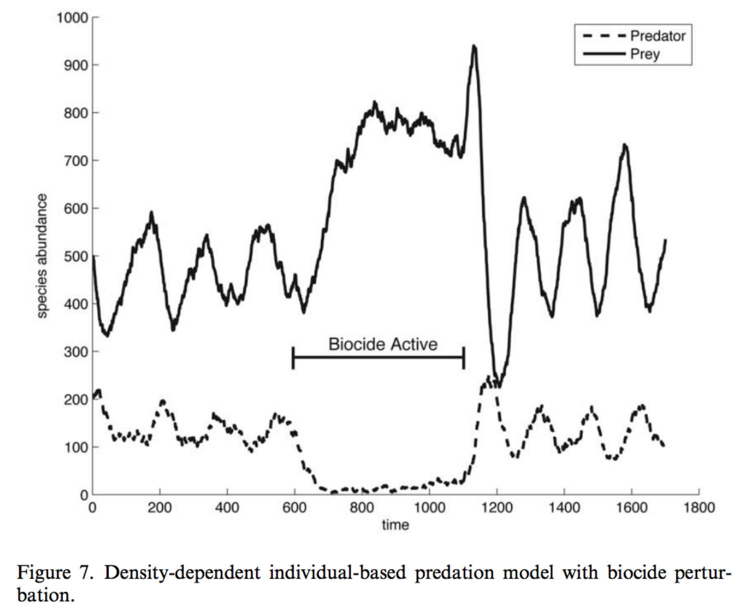


Figure 5. Simple individual-based model corresponding to the Lotka-Volterra model. In this trial, both species go extinct.

Weisberg and Reisman however shows that after a number of adjustments, e.g., the specification of further parameters such as the carrying capacity, the system starts exhibiting regular oscillations, at least temporarily. At this point, it is possible to show that by introducing an external cause of death in the system, the Volterra Principle holds true in IBMs too (see figure 3). The key to get to the Volterra Principle is that the two systems share an underlying structure, namely a negative-coupling between the two species. Both models describe the coupling in different ways, however, that is all that is needed to get to a robust result.



(a) Picture 1 (b) Picture 2



3 Diachronic and Synchronic Structure Preservation

In this section, we present our interpretation of many-models robustness in terms of structural realism. Our claim is that robustness across different models, like in the case of the Lotka-Volterra principle, is evidence for the result being (approximately) true of a shared structure among the models. In order to properly articulate this claim, let us begin by saying a few words on structural realism in general.

Structural realism is the view that our best scientific theories are (approximately) true of the structure they describe.[2] Structural realism was introduced into the debate by Worrall (1989) as a response to Laudan's *pessimistic (meta-)induction* against scientific realism (Laudan 1981). Take *scientific realism* to be the view that our best scientific theories are (at least approximately) true. Laudan's argument roughly goes as follows:

Pessimistic Induction Most of the best scientific theories of the past have turned out false. Therefore, also the best scientific theories of the present are likely false.

This is a *prima facie* compelling argument. Its premise seems like an indisputable fact of the history of science, confirmed by a quick look around the junkyard of scientific theories. And the inference rule is enumerative induction, a principle held dear by scientific realists.

Worrall aims to undermine the argument by challenging its premise. According to him, while there is a sense in which the premise is true, it is an unfortunate oversimplification. Strictly speaking, so Worrall claims, many old theories, like Newtonian mechanics, say, have been superseded by new theories, in Newton's case by Einstein's theory of general relativity. And in fact, there is a sense in which Newton's theory has been proven false by Einstein: the claims of general relativity are *inconsistent* with the claims of Newtonian mechanics. Moreover, Newtonian mechanics is formulated in a framework of absolute space and time, which is incompatible with the now generally accepted relativistic framework of four-dimensional spacetime.

But Worrall points out that it is not like the old theories were thrown out completely. Essentially, he argues that there is a "kernel of truth" in Newton's theory, which is preserved in general relativity. Worrall bases this assessment on the observation that Newton's laws are still *approximately* true in general relativity, given that we are dealing with relatively low speeds compared to the speed of light. Consider momentum as an example. According to Newton, if m is an object's mass and $\mathbf{v}$ its velocity, then its Newtonian momentum, $\mathbf{p}_{\text{Newton}}$, is:

$$\mathbf{p}_{\text{Newton}} = m\mathbf{v}.$$

In general relativity, in contrast, the relativistic momentum, $\mathbf{p}_{\text{Einstein}}$, is:

$$\mathbf{p}_{\text{Einstein}} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}.$$

[2] For an overview of the literature, see (Ladyman2016).

Now note that if $v \ll c$, we get that $\mathbf{p}_{\text{Newton}} \approx \mathbf{p}_{\text{Einstein}}$, since then we get that $\frac{\mathbf{v}^2}{c^2} \approx 0$ and so $\sqrt{1 - \frac{\mathbf{v}^2}{c^2}} \approx 1$. Note further that this really is an approximation, since *only* for $\mathbf{v} = 0$, we get a strict identity.

Based on this assessment, Worrall argues that there is a continuity of form or *structure* from Newton to Einstein. This leads him to endorse structural realism: what the theories in question are about is not the world itself, but rather its structure; and this is what they truly describe. So the sense in which the premise of the pessimistic induction is wrong according to Worrall is that even though Newton was abandoned and is now considered strictly speaking false, Newtonian mechanics still makes approximately true claims about the structure it describes. This structure is what is preserved across theory change and is what the scientific realist ought to be interested in.

Today, structural realism is generally considered the most defensible form of scientific realism (Ladyman 2016). The view comes in many shapes and forms and has been applied to many different sciences. In the following, however, we do not want to get bogged down with the details of the different formulations of structural realism, but simply wish to work with the basic ideas of the view.[3] In particular, we will work with the following three assumptions:

Structural Aboutness. The theories of most mature sciences are about structure.

Structural Realism. Our best scientific are approximately true of the structures they describe.

Structural Continuity. Typically, what is preserved across theory change is (part of) the structural subject matter and (at least some) true claims about that structure.

We also do not aim to consider all the possible applications of structural realism in science. For the present purpose, we restrict ourselves to biology, more specifically population dynamics, from which we recruited our example of many-models robustness. Note, however, that French (French 2011) has argued that structural realism provides a good interpretation of theory change and scientific practice in biology.

Note also that in the background story for structural realism sketched above, structural continuity is used in a *diachronic* sense: an old theory, which has been discarded, is

[3]Most of what we're going to say can be spelled out in more detail on most variants and specifications of structural realism.

structurally continuous with a new theory, which is now endorsed. But nothing about the concept of structural continuity forces us to restrict ourselves to diachronic applications. Nothing stops us from considering structural continuity in a *synchronic* sense, i.e. as holding between two scientific theories (or models) that are both currently endorsed. This becomes particularly interesting when we are considering two theories (or models) whose assumptions conflict, as in the case of the Lotka-Volterra model and IBMs. As we have pointed out in the previous section, although the Lotka-Volterra model and IBMs make similar predictions about population dynamics—in particular, both models validate the Volterra principle—the math of the two models is fundamentally different, to the extent that they conflict in their fundamental assumptions. Our aim is to provide an interpretation of the situation in terms of structural continuity.

The situation is really quite analogous to the case of Newton and Einstein. Just like Newtonian mechanics is formulated in a framework of absolute space and time while general relativity is formulated in a framework of four-dimensional spacetime, the Lotka-Volterra model is formulated in a framework of continuous population growth while IBMs are formulated in framework of discrete population growth. And, what we will argue next, just like there is a structural continuity from Newton to Einstein, there is a structural continuity from Lotka-Volterra models to IBMs. In order to make our point, let us take a moment and think about the concept of structural continuity.

For Worrall, structural continuity is found at the level of the mathematical equations of the theories (or models) in question. Sometimes, we find a complete continuity, as in the case of the move from Fresnel's wave theory of light to Maxwell's electromagnetic theory, which is discussed in detail by Worrall. In such a case, the old equations are simply given a new interpretation in the new theory. But most commonly, what we find is that the equations of the new theory yield the equations of the old theory as certain limiting cases, like in the case of Newton and Einstein discussed above.

Even though a precise formulation of structural realism could be clarified further-as Worrall himself points out (Worrall 1989, p. 161)–for the present purpose, it gives us enough to determine a sufficient criterion for structural continuity:[4]

Structural Continuity. If sufficiently much of the mathematics of theory (or model) T_1

[4]See, for example, (Ladyman 2007) for ways of spelling out the concept of structure and structural continuity in more detail.

can be recovered in theory (or model) T_2, e.g. by recovering the equations of T_1 as limit cases of equations in T_2, then there is structural continuity from T_1 to T_2.

We will use this criterion to establish that there is a structural continuity from IBMs to LVE models. Consider a concrete run of the IBM model, giving us a data set consisting of a prey population $x(t)$ and a predator population $y(t)$ for each time point t in the (discrete) interval $I \subseteq \mathbb{R}$ of the simulation run. Since, by design, the simulations exhibit relatively stable oscillations, we find a subinterval $I' \subseteq I$ during which $x(t)$ and $y(t)$ behave like two negatively coupled periodic functions. More specifically, glossing over some details, we will be able to find a pair of solutions to a set of Lotka-Volterra equations $x'(t)$ and $y'(t)$, which are defined over the whole set $\mathbb{R}$ of real numbers and minimize the (squared) distances $|x(t) - x'(t)|^2$ and $|y(t) - y'(t)|^2$ for all data points $t \in I'$.[5] In fact, by a concrete data analysis, we are able to show that these distances are "objectively" small. In other words, IBM simulation runs discretely approximate solutions to the Lotka-Volterra equations.

By the principle **Structural Continuity**, this establishes that there is a structural continuity from IBMs to Lotka-Volterra models. But how does this continuity address the issue of many model robustness? We claim, in the spirit of structural realism, that the structural continuity between IBMs and Lotka-Volterra models suggests that they are models of one and the same *structure*: the structure of negatively coupled prey-predator dynamics with (relatively) stable oscillations. The two models instantiate this structure in very different ways but there is a shared form between them. The fact that the Volterra principle holds in both models, then, is evidence that it is (approximately) true of their shared structure. So, according to our structural realist interpretation that is what is going on in cases of many-models robustness: scientists are using different models to represent the same underlying structure in wildly different ways and establish that the structure itself has a property by showing that the property is invariant across these models.

[5] More specifically, we minimize $\sum_{t \in I'} |x(t) - x'(t)|^2$ and $\sum_{t \in I'} |y(t) - y'(t)|^2$.

4 Conclusion

Many-models robustness is a puzzling phenomenon. Why are scientists interested in results being invariant across fundamentally different models? We have proposed an answer in terms of structural realism: what the scientists are showing is not that the concrete systems they are describing has the property but rather that a common structure across these models has the property. This interpretation is in line with a diachronic, pluralist picture of structural realism, according to which scientific theories and models are approximately true of the structures they describe and, additionally, there may be more than one way of describing the same structure.

We wish to suggest that this opens up a new interesting line of research in structural realism. The project would be to identify further cases of multiple models robustness and show that they share a common structure, in a similar way as we did in the case of IBMs and Lotka-Volterra models. Furthermore, the project has the potential to shed light on a wide range of issues, like reasoning by analogy in physics, as discussed in (Dardashti, Thébault, and Winsberg 2017), for example. Overall, these projects would shift the focus from historic theory change to contemporary theory (or model) diversity.

(3862 words including references + 4 diagrams/figures)

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