Tableaux for exact entailment

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Conjunction

- if $\mathbf{M}, s \vDash \bigwedge \Gamma_1, \ldots, \mathbf{M}, s \vDash \bigwedge \Gamma_n$, then $\mathbf{M}, s \vDash \bigwedge \bigcup \{\Gamma_1, \ldots, \Gamma_n\}$;
- if $\Gamma \subseteq \Delta \subseteq \Sigma$, $\mathbf{M}, s \vDash \bigwedge \Gamma$, and $\mathbf{M}, s \vDash \bigwedge \Sigma$, then $\mathbf{M}, s \vDash \bigwedge \Delta$

Rules

- $\Gamma, +$ means $\mathbf{M}, s \vDash \bigwedge \Gamma$;
- $\Gamma, -$ means $\mathbf{M}, s \nvDash \bigwedge \Gamma$.

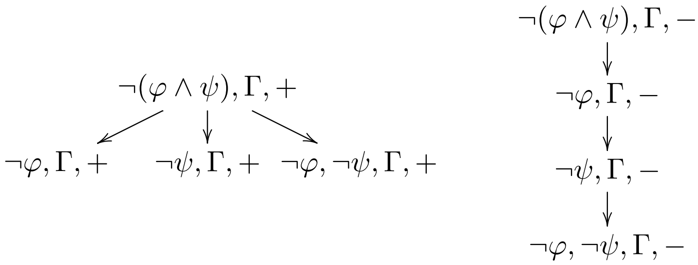
$$\varphi \vee \psi, \Gamma, + \quad \longrightarrow \quad \varphi, \Gamma, + \quad \mid \quad \psi, \Gamma, + \quad \mid \quad \varphi, \psi, \Gamma, +$$

$$\varphi \vee \psi, \Gamma, - \;\downarrow\; \varphi, \Gamma, - \;\downarrow\; \psi, \Gamma, - \;\downarrow\; \varphi, \psi, \Gamma, -$$

$$\varphi \wedge \psi, \Gamma, + \;\downarrow\; \varphi, \psi, \Gamma, + \qquad \varphi \wedge \psi, \Gamma, - \;\downarrow\; \varphi, \psi, \Gamma, -$$

$$\neg\neg\varphi, \Gamma, + \;\downarrow\; \varphi, \Gamma, + \qquad \neg\neg\varphi, \Gamma, - \;\downarrow\; \varphi, \Gamma, -$$

$$\neg(\varphi \vee \psi), \Gamma, + \;\downarrow\; \neg\varphi, \neg\psi, \Gamma, + \qquad \neg(\varphi \vee \psi), \Gamma, - \;\downarrow\; \neg\varphi, \neg\psi, \Gamma, -$$



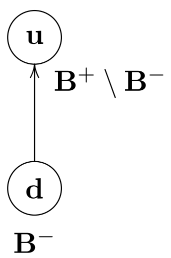
Closure rule

A branch closes iff there are Γ, Δ, such that:

- $\Gamma, + \in B$;
- $\Delta, - \in B$;
- $\Gamma \subseteq \Delta \subseteq \bigcup_{\Sigma, + \in B} \Sigma$.

Countermodel for an open branch

- $\mathbf{B}^- = \bigcup\{\Gamma \subseteq \Lambda : \Gamma, - \in B \text{ and } \exists\Delta(\Delta, + \in B \text{ and } \Gamma \subseteq \Delta)\}$
- $\mathbf{B}^+ = \bigcup\{\Gamma \subseteq \Lambda : \Gamma, + \in B\}$



$$S_B = \{\mathbf{u}, \mathbf{d}\}$$

$$\sqsubseteq = \{\langle \mathbf{d}, \mathbf{d} \rangle, \langle \mathbf{d}, \mathbf{u} \rangle, \langle \mathbf{u}, \mathbf{u} \rangle\}$$

$$V^+(p) = \begin{cases} \mathbf{d} & \text{if } p \in \mathbf{B}^- \\ \mathbf{u} & \text{if } p \in \mathbf{B}^+ \setminus \mathbf{B}^- \end{cases}$$

$$V^-(p) = \begin{cases} \mathbf{d} & \text{if } \neg p \in \mathbf{B}^- \\ \mathbf{u} & \text{if } \neg p \in \mathbf{B}^+ \setminus \mathbf{B}^- \end{cases}$$

References

[1] Fine, Kit. 2017. "Truthmaker Semantics." In: *A Companion to the Philosophy of Language*, 2nd edition, edited by Bob Hale, Crispin Wright, and Alexander Miller, 556–77. New York, NY: Wiley.

[2] Fine, Kit and Mark Jago. 2017. "Logic for Exact Entailment." *Unpublished Manuscript*.